DIFFERENTIAL-GEOMETRY SCALING METHOD FOR ELECTROMAGNETIC FIELD AND ITS APPLICATIONS TO COAXIAL WAVEGUIDE JUNCTIONS*

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Summary

It is well-known that in mechanics and fluid dynamics one can transform or scale one problem and its solution to create a whole class of equivalent problems and their solutions[1]. Different problems and their solution behaviors of one equivalent class may look very different, but among them there are properties they share. The essence of such a scaling is to get appropriate dimensionless parameters that are common to them all.

However, in electromagnetic (EM) theory the nature and application of such a similar scaling method[2], except for conformal mappings of static fields, has not been given extensive attention. Only a few articles have recently been devoted to it[3]. The purpose of this work is to thoroughly investigate and develop the nature, the limitation, the usefulness, and the application of such a scaling method for EM theory by using differential-geometry approach.

We first explore and develop the general framework of the method that can carry an EM problem P of complicated geometry into an equivalent problem P' of simple Cartesian or other simple geometry with its accompanying transformations for medium, geometry, source, and field. The advantage of such a procedure is, hopefully, to make the complexities of the geometry and of the medium "cancel" each other in such a way that the resulting problem is simple and solvable. Then we obtain various conditions and limitations of the method as imposed by special choices of geometry, medium, and field modes. These include time independent scaling, orthogonal coordinates, diagonal media, and especially emphasized coaxial systems.

We then present an application of this differential geometry scaling method to two specific problems. In one a perfect matching section between a cylindrical and a conical coaxial waveguide is obtained by appropriately loading the section with inhomogeneous μ and ε, and all relevant electromagnetic quantities and geometrical boundaries are tabulated. In the other a perfect matching section between two cylindrical coaxial waveguides is found with the appropriately shaped matching section loaded by inhomogeneous ε, anisotropic conductivity $\underline{\underline{\sigma}}$, and constant μ. All results are tabulated and plotted.

Finally we noticed that the parallel-plate Cartesian scaled version of the fixed μ matching may give the fixed μ matchings of other geometrical shapes by permitting some variances in its $P' \rightarrow P$ scaling procedure. This aforesaid aspect, the criterions for a continuous tapered matching section, and the non-orthogonal scaling which can make use of the Brewster angle transmission in a natural way are discussed for work of future interest.

Footnotes and References

* Part of this work has been published by T. C. Mo, C. H. Papas, and C. E. Baum in the J. of Math. Phys. 14, 479 (April 1973). Also this report is mostly based on EMP Sensor and Simulation Note 169, (March 1973) by these same authors. Also this work was partly supported by the Air Force Weapons Laboratory.

1. See, e.g., L. D. Landau and E. M. Lifshitz, Fluid Dynamics, Pergamon Press (1959), Secs. 19, 53, 118, 119; also, Mechanics, Pergamon Press (1960), Sec. 10; for an earlier example of such transforms, see E. J. Routh, Proc. London Math. Soc. 12, 73-89 (1881).

2. Some different examples of the electromagnetic scaling can be found in the following. For conformal mapped waveguide see, e.g., F. E. Borgnis and C. H. Papas, in Handbuch der Physik, Springer-Verlag, Berlin (1958) 16, 358; also F. J. Tischer, (Correspondence) Proc. IEEE 51, 1050 (July 1963), and (Correspondence) Proc. IEEE 53, 168 (Feb. 1965); also J. A. Stratton, Electromagnetic Theory, McGraw-Hill (1941), 217; also P. Krasnooshkin, J. Phys. USSR 10, 434 (1946); for a frequency scaling of reflection see, J. H. Davis and J. R. Cogdell, IEEE Trans. AP-19, 58 (Jan. 1971); for a scaling for reducing constantly moving uniform simple media, see R. J. Pogorzelski, IEEE Trans. AP-19, (Communication) 455 (May 1971).

3. C. E. Baum, EMP Sensor and Simulation note 32 (Jan. 1967) and note 64 (1968); see also P. L. Uslenghi, IEEE Trans. AP-17 (Communication), 644 (Sept. 1969).